The longitudinal and lateral distributions of interference for such a configuration, $P_t = 0.27$ and $P_b = 0.70$, are plotted in Figs. 3 and 4. These curves indicate that the upwash interference is reduced to practically zero in both the streamwise and spanwise direction from the model. Some results[3] of a finite span model in the scheduled height-to-width closed-on-bottom tunnels are shown in Figs. 3 and 4 for comparison purposes.

In conclusion, a selected single configuration was obtained which eliminates interferences, in the streamwise and spanwise direction simultaneously, for all rotor wake skew angles.

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A Rapid Method for Flow Splitter Design in 3-D Exhaust Nozzles

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Nomenclature

A = flow area
H = flow height
l = circular arc length
L = nozzle length
m = number of channels
M = Mach number
p = pressure
r = radial coordinate
T = temperature
W = flow width
z = axial coordinate
α = area fraction
ρ = mass density
ϕ = circumferential coordinate

Subscripts

B = bifurcation
I = inner
j = circumferential boundary index
k = axial coordinate index
L = lower
M = mean
O = outer
s = stagnation
T = total
U = upper

Superscripts

j = jth circumferential boundary

I. Introduction

THE design of flow splitters in exhaust nozzles is a problem which arises in the design of transport jet engines that cruise at high subsonic Mach numbers. Flow splitters are included in exhaust nozzles to provide surface area for acoustical treatment and to reduce swirl. Hence, the annular passage of the exhaust nozzle is divided into a number of circumferential channels by radial splitters. The design problem consists of determining the splitter locations such that the individual channels have similar flow distributions. The approach of the present design method is to compute the flow area distribution in each channel by a three-dimensional flow area approximation. Then, based on the assumption of a one-dimensional steady compressible inviscid flow, the corresponding Mach number distributions are obtained in each channel. The channel walls are constructed in such a manner that the one-dimensional Mach number distributions in each channel are identical. The present design method is easily programed on a digital computer and has been found to be quite useful for radial splitter design in fan exhaust nozzles used in high-bypass-ratio jet engines.

In this paper, a design method for flow splitters in three-dimensional fan exhaust nozzles is presented. The nozzle contours are represented by surfaces of revolution, and the flow passage is divided into a given number of channels by radial splitters. The basic assumption of the present design method is that the flowfield in each channel of the exhaust nozzle can be considered as a one-dimensional steady compressible flow. Then, based on the conformal representation of an arbitrary surface of revolution by a right circular cylinder[1] an approximation for the channel flow area distributions can be computed in order to represent the nozzle flowfield by one-dimensional equations. A similar idea for the calculation of flow area was proposed by Pratt and Whitney[2]; however, their procedure was graphical in nature. The design problem envolves determining the circumferential position of the radial flow splitters in such a manner that the Mach number distributions in each of the channels are identical.

II. Nozzle Geometry

A cylindrical coordinate system (r,ϕ,z) is taken to describe the fan exhaust nozzle (Fig. 1). The upper and lower pylon that bifurcate the nozzle are accounted for by defining so called upper and lower bifurcation angles ϕ_U and ϕ_L, whose variation with the z coordinate is assumed to be given. Hence, the region available for mass flow consists of two symmetric channels. Therefore, it is sufficient to consider the region defined by

$$\begin{gathered} r_I(z) \le r \le r_0(z) \\ \phi_L(z) \le \phi \le \pi - \phi_U(z) \\ 0 \le z \le L \end{gathered} \tag{1}$$

If the region defined by Eq. (1) is divided into m channels by $(m - 1)$ radial splitters, the angles $\phi_j(z)$, $j = 1,2, \ldots, m + 1$ define the circumferential channel boundaries (Fig. 1).

III. Flow Area Approximation

The definition of flow area in the jth channel is based on the concepts of a flow width $W_j(z)$ and a flow height $H(z)$. If the mean radius distribution is defined by the arithmetic mean

$$r_M(z) = \tfrac{1}{2}[r_I(z) + r_0(z)],\ 0 \le z \le L \tag{2}$$

then the flow height is defined by

$$H(z) = [r_0(z) - r_I(z)] \cdot \cos[\tan^{-1}(r'_M)] \tag{3}$$

where the prime denotes differentiation with respect to z. It should be noted that the mean radius distribution introduced in Eq. (2) defines a mean surface of revolution $[r_M(z), \phi, z]$. Hence, the flow area in the jth channel is expressed by

$$A_j(z) = W_j(z) \cdot H(z),\ j = 1,2, \ldots, m \tag{4}$$

The flow width $W_j(z)$ in the jth channel is defined as an arc

Received January 15, 1970.

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which lies on the mean nozzle surface with the property that it is orthogonal to the circumferential boundaries of the jth channel. The exact manner in which an arc having this property is derived will now be discussed.

Since it is shown in Ref. 1 that any surface of revolution can be represented conformally by the surface of a right circular cylinder which, in turn, can be represented isometrically by a plane, then the mean nozzle surface defined by Eq. (2) can be represented by a plane.

The equations which define the one-to-one correspondence between the mean nozzle surface and its plane representation are given by

$$l_j(\bar{z}) = r_M(0)\cdot\bar{\phi}_j = r_M(0)\cdot\phi_j, \quad j = 1,2,\ldots,m+1 \tag{5a}$$

$$\bar{z} = \int_0^z \frac{r_M(0)}{r_M(\bar{z})}[1 + (r_M')^2]^{1/2}d\bar{z} \tag{5b}$$

where the bar denotes variables on the transformed plane surface, and $\bar{z}$ is a dummy variable of integration.

The mean nozzle surface is now represented by the region in the $l - \bar{z}$ plane defined by (Fig. 2)

$$r_M(0)\cdot\phi_L \leq l \leq r_M(0)\cdot(\pi - \phi_U) \tag{6a}$$

$$0 \leq \bar{z} \leq \bar{L} \tag{6b}$$

The curves $l_j(\bar{z})$, $j = 1,2,\ldots,m+1$ represent the circumferential channel boundaries.

It is now possible to construct circular arcs which are orthogonal to the circumferential channel boundaries in the transformed plane. For computational purposes, the orthogonal construction is made at discrete points $(l_{j,k};\ \bar{z}_k)$, $j = 1,2,\ldots,m+1$; $k = 1,2,\ldots,n$. The relation between $l_{j+1,k}$ and $l_{j,k}$ on a typical circular arc is given by

$$\frac{l_{j+1,k} - l_{j,k}}{\bar{z}_k^{(j+1)} - \bar{z}_k^{(j)}} = \frac{\cos\theta_j + \cos\theta_{j+1}}{\sin\theta_j + \sin\theta_{j+1}} \tag{7}$$

where $\theta_j = \tan^{-1}(l'_{j,k})$ and $\theta_{j+1} = \tan^{-1}(l'_{j+1,k})$. Given $(l_{j+1,k};\ \bar{z}_k)$, Eq. (7) is solved iteratively for $(l_{j,k};\ \bar{z}_k)$. The flow width is defined as the arc length of the corresponding circular arc on the actual mean nozzle surface. Hence,

$$W_{j,k} = \int_{\bar{z}_k^{(j)}}^{\bar{z}_k^{(j+1)}} \frac{r_M(\bar{z})}{r_M(0)}[1 + (l')^2]^{1/2}d\bar{z} \tag{8}$$

$$j = 1,2,\ldots,m; \quad k = 1,2,\ldots,n$$

It should be noted that because of the conformal property, the flow widths are orthogonal to the radial splitters on the actual mean nozzle surface. The total flow area distribution

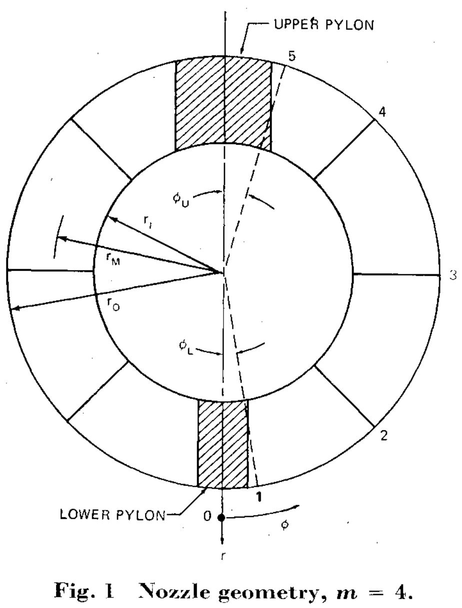

Fig. 1 Nozzle geometry, $m = 4$.

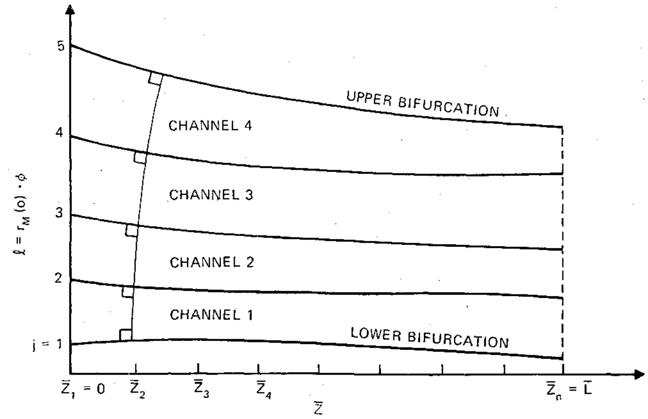

Fig. 2 Plane representation of mean nozzle surface with bifurcation, $m = 4$.

of the nozzle is obtained from Eq. (4) as

$$A_T(z_k) = 2\sum_{j=1}^{M} A_j(z_k), \ k = 1,2,\ldots,n \tag{9}$$

IV. Annular Nozzle

For the special case of an annular nozzle (zero pylon width) it is easily shown that the area approximation given by Eq. (9) reduces to the lateral surface area of the frustum of a cone which is a commonly used approximation for flow area in annular nozzles. The simplification lies in the fact that without a pylon the plane representation consists of horizontal lines in the $l - \bar{z}$ plane. The orthogonal construction consists of vertical lines in the $l - \bar{z}$ plane (circular arcs of infinite radii). Hence, Eq. (8) simplifies to

$$W_{j,k} = r_M(z_k)\pi/m \tag{10}$$

Therefore, the total annular flow area is given by

$$A_T(z_k) = 2\pi r_M(z_k)\cdot[r_O(z_k) - r_I(z_k)]\cdot\cos[\tan^{-1}(r_M')] \tag{11}$$

Equation (11) represents the lateral surface area of the frustum of a cone.

V. Flowfield

For a given set of splitter angles $\phi_j(z)$, $0 \leq z \leq L$, $j = 1,2,\ldots,m+1$, the channel flow area distributions can be calculated from Eq. (4). If it is further assumed that each channel is choked at its respective exit area, the one-dimensional inviscid compressible flow variables are easily computed from

$$\frac{A_j(z)}{A_j(L)} = \frac{1}{M_j}\left\{\frac{2}{(\gamma+1)}\left[1 + \left(\frac{\gamma-1}{2}\right)M_j^2\right]\right\}^{(\gamma+1)/2(\gamma-1)} \tag{12a}$$

$$(p/p_s)_j = \{1 + [(\gamma-1)/2]M_j^2\}^{-\gamma/(\gamma-1)} \tag{12b}$$

$$(T/T_s)_j = \{1 + [(\gamma-1)/2]M_j^2\}^{-1} \tag{12c}$$

$$(\rho/\rho_s)_j = \{1 + [(\gamma-1)/2]M_j^2\}^{-1/(\gamma-1)} \tag{12d}$$

VI. Design Problem

It is clear from the discussion in the previous sections that for a given set of splitter angles, the one-dimensional flowfield in each channel can be computed. However, for arbitrary choices of ϕ_j, $j = 2,3,\ldots,m$ the Mach numbers (static pressures) at a given z location in the various channels will not be equal. Therefore, if the splitters are terminated before the nozzle exit (which is usually the case), then an undesirable flow adjustment can take place. In order to avoid this situation, it is desirable to choose the splitter angles ϕ_j, $j = 2,3,\ldots,m$ in such a manner that the one-dimensional flows at corresponding z locations in each channel have the same Mach number (static pressure).

If it is assumed that the splitter angles are prescribed at the entrance to the nozzle, then the remaining splitter angles are determined from the condition of identical one-dimensional flows. For convenience, let

$$A_j(z) = \alpha_j(z) \cdot A_T(z), j = 1,2, \ldots, m \tag{13}$$

where $\alpha_j(z)$ are the appropriate fractions which relate the channel flow areas to the total flow area. Since $\alpha_j(0)$, $j = 1,2, \ldots, m$ are given, $\alpha_j(L), j = 1,2, \ldots, m$ are found from the solution of the linear system

$$\begin{bmatrix} \alpha_2(0) - \alpha_1(0) & 0 & \ldots & 0 \\ \hdashline 0 & \ldots & 0 & \alpha_m(0) - \alpha_{m-1}(0) \\ 1 & \ldots & 1 \quad 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1(L) \\ \hdashline \alpha_{m-1}(L) \\ \alpha_m(L) \end{bmatrix} = \begin{bmatrix} 0 \\ \hdashline 0 \\ 1 \end{bmatrix} \tag{14}$$

It is easy to show that

$$\alpha_j(L) = \alpha_j(0), j = 1,2, \ldots m \tag{15}$$

In a similar manner

$$\alpha_j(z) = \alpha_j(0), j = 1,2, \ldots, m; \quad 0 \leq z \leq L \tag{16}$$

If the splitter angles are chosen in this manner, the one-dimensional flows in each channel are identical.

VII. Conclusion

A design method for radial flow splitters in three-dimensional fan exhaust nozzles has been presented. With the proper choice of splitter angles, identical one-dimensional flows can be obtained in each channel. These design criteria avoid undesirable flow adjustments at the splitter exits which could lead to a considerable loss of thrust. This design procedure is easily programed on a digital computer so that rapid design calculations are easily carried out. The generalization to nozzles with circumferential as well as radial splitters is also possible.

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